

How Percepts and Concepts Engage the Future

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Abstract With Humean empiricism and its agnostic stance regarding the future as a foil, I take a bird's eye view of the links between past and future prescribed by ordinary concepts of everyday things and processes, and by scientific models of phenomenal situations. I argue that they entitle us to claim knowledge of the future—including, where appropriate, its necessary course—in a humanly affordable sense of 'knowledge'.

Keywords Induction · Probability · Necessity · Knowledge of the future

Bruno de Finetti held that we cannot know the future; at most, we can bet on it.¹ *Previsions* (in the peculiar sense he gave this word), a subset of which are probability statements, have nothing to do with foreknowledge² and should never be confused with *predictions*. De Finetti took this stance squarely within the Humean tradition, which ultimately would ground all knowledge on sense impressions. Since future impressions have yet to be and present impressions turn out to be past before one even realizes their presence, from a Humean standpoint knowledge properly rests on memory alone. However, the enlightened Kantian enjoys a different view. He is well aware that raw sense impressions are available to us only in the interstices of a conceptual network, to which we promptly tie them up as well as we can. Unconnected sense impressions are perhaps the stuff dreams are made on, but they are not to be

¹ Bruno de Finetti's philosophical stance is further discussed in the Supplementary Remarks at the end of this paper.

² Kyburg's rendering of *prévision* as 'foresight' in the title of de Finetti [5], egregiously confirms the proverbial Italian saying *traduttore traditore* (cf. [18, 53]).

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had in wakeful life. In particular, observation, i.e. attentive perception, which is the mainstay of empirical knowledge, is always perception of *something* as *something*, and thus of an *object* under a *concept*, which binds its many diverse jointly and successively displayed sensory aspects under a rule of composition.³ Now concepts of present objects often engage the future and give us a grip on what is about to happen.

This cognitive power of concepts can be readily appreciated in the perception of everyday things. When I see a glass of red wine on the dinner table in front of me I unhesitatingly expect to feel the sure grasp of my fingers on the cool hard stem of the glass when I seize it, to smell the bouquet as I bring the glass close to my nose, to savor the taste of red wine as I sip some with my mouth. It is worth noting that I do not simply expect the repetition of previously experienced sense impressions whose image is entrenched in my mind, as a Humean would have it; for all I know, the glass might contain a most peculiar red wine from Australia, different from every wine I've tasted, and still meet my expectation. My concept of a glass of red wine encompasses a rich spectrum of tastes, with ample room for the unknown, but certainly excludes the taste of Chardonnay, not to mention Coca-Cola and colored water. Should the wine in the glass in front of me smell or taste like water, my expectation would be frustrated and I would reach for a different concept under which to collect my percepts.

I shall explore a few examples from science that illustrate the functioning and the span of concepts in the constitution of our often mistaken yet unmistakable knowledge of the future. But before going into it, I shall briefly take a Kantian's look at probability, which differs significantly from the Humean view proposed by de Finetti. The Kantian approach involves regarding—that is to say, conceiving—certain present situations as *chance setups*, a concept that is not clearly documented before the 17th century,⁴ but which has since taken over our basic understanding of ever more varied and complex realities. To effectively view a particular process as the successive production of outcomes by a definite chance setup amounts to expecting (i) that those outcomes will vary over a definite range of alternatives that make up the “probability space” and (ii) that the relative frequency of each alternative among the total number of outcomes will tend to some definite limit—assigned to that alternative by the “probability function”—as the number of outcomes increases indefinitely. As in the case of the glass of wine, the failure of such expectations leads to a revision of the concept, either by modifying the composition of the probability space or the values assigned to each element of it by the probability function, or, more drastically, by wholly discarding the idea that we are in presence of a chance setup. While the initial concept is upheld it furnishes us with a *purported* knowledge of the future, not to be confused with a groundless guess. On occasions it may be just a personal

³Kant was fond of saying *synthesis*, in Greek, but he also used the German word *Zusammensetzen* (‘composition’). See, for instance, Kant [17, 12: 222f.; 20: 275f]. In [17, 20: 271] he describes his renowned categories of the understanding as “Arten der Zusammensetzung (Synthesis) mit Bewußtsein”.

⁴Galileo's paper “Sopra le scoperte dei dadi” preceded by several decades the well-known letter from Pascal to Fermat (29.7.1654; see [23, 1: 146–162]) and Huygens' “Van Rekeningh in Spelen van Geluck”. This was the first treatment of the subject to appear in print; it was mailed to van Schooten on 20.4.1656, and published in Latin translation by the latter in 1657 as *De ratiociniis in aleae ludo* (reproduced in [2]). Galileo's paper was not published until 1718; it can now be read in Galileo [14, 8: 591–594] (English translation by E.H. Thorne in <http://www.leidenuniv.nl/fsw/verduin/stathist/galileo.htm>).

feeling or a wilful bet that such and such a situation *is* a chance setup describable in terms of probability theory in such and such a way. (E.g. when one decides how much to invest on drilling for oil in a particular piece of deep ocean bottom). But the quantitative implications of this description are surely knowledge—and quite precise knowledge too—in any plausible meaning of this word (which in its ordinary use normally connotes a good number of hypothetical conditions). Evidently, such plausible meanings differ widely from the premodern sense of ‘knowledge’ embedded in Plato’s overbearing notion of *episteme*,⁵ which led the more lucid intellectuals to Pyrrhonism, while the rest took refuge in one or the other of the rigid dogmatic belief systems of late Antiquity and the Middle Ages.

To conceive a complex situation as a chance setup is a fairly sophisticated instance of *modeling*, the characteristic modern form of scientific grasp of perceivable realities, to which I now turn. It deserves notice—before I proceed any further—that when, say, a roulette table together with its croupier is conceived as a chance setup one need not ignore the basic model of roulette as a classical deterministic dynamical system, but ought rather to assume it; chance inexorably enters the picture when we see the croupier blindly picking initial conditions as if from an urn.⁶

To my knowledge, the first modeling of a phenomenal process was proposed by Eudoxus of Cnidus, a younger contemporary of Plato.⁷ He modeled the motion of the “planets” or wandering stars (πλανήτες, celestial light sources that do not keep constant distances to the fixed stars) by conceiving each one of them as a brilliant point fixed on a sphere that rotates uniformly about an axis fixed on a second sphere, homocentric with the former, which rotates uniformly about an axis fixed on a third sphere, homocentric with the former. . . , which rotates uniformly about an axis fixed on an n -th sphere, homocentric with the former, which is the firmament or sphere of the fixed stars (rotating uniformly about the celestial poles). Like all great ideas, it is quite likely that the Eudoxean planetary models had precursors, but I am not aware of them. What matters to us is that they distinctly anticipate in definite ways the epistemic practices of modern natural science.

In the light of a planet’s past motion among the fixed stars, Eudoxus constructs a kinematic model that prescribes its future motions. The model *ignores* several properties of the phenomenal planet—viz. its color, its apparent size and luminosity—but does not *deny* them. For the sake of ascertaining the declination and right ascension of the planet at any given time, those properties are simply irrelevant. The n -sphere model of a planet depends on the $3(n - 1)$ parameters that determine the angular velocity vector of each sphere except the firmament. The model did its job insofar

⁵Remember that Plato’s spokesman “Socrates”, in *The Republic*, Book VII, explicitly excludes from *episteme* every branch of mathematics existing at that time, besides all the many forms of dexterous understanding of some particular field of experience that made someone an ἐπιστήμων in the established meaning of this word in Attic Greek. Plato’s *episteme* has remained forever an unpaid promissory note.

⁶One might perhaps with some effort succeed in incorporating roulette, ball, croupier, players and much else into a vast, fairly closed deterministic system. But this would be an utterly different object from the roulette tables one encounters in casinos and understands as chance setups.

⁷Only fragments survive of Eudoxus’ scientific writings; they were edited by Laserre [12]. Modern elucidations in Schiaparelli [32]; Dreyer [8, 87–107]; Duhem [9, 1: 102–123]; Jacobsen [16, 34–40]; Torretti [33, 35–42].

as it empowered the Greek astronomer to compute the motion of the planet with the mathematical resources available to him. We now know—since Fourier—that by increasing n and improving the angular velocity parameters one could in principle approach the actual trajectory of each planet on the surface of the firmament as closely as one wished. Eudoxus' follower Calippus took a first step in this direction. Even with this correction, Eudoxus' predictions of planetary positions must have been pretty inaccurate by modern standards; but their modicum of success sufficed to convert Plato from a derider into an enthusiast of physico-mathematical astronomy.⁸ Coarser minds bestowed on Eudoxus' spheres a corporeal presence, regarding them as extremely thin nested balls made of some exceedingly malleable frictionless material unknown on Earth. This approach generated the labyrinthine heavens of Aristotle (*Metaphysica* Λ , 8), an achievement that—whatever one might think of its intrinsic merit—put physical cosmology at odds with mathematical astronomy as soon as the Greek astronomers, seeking to improve the accuracy of their predictions, opted for eccentric spheres and epicycles (which made allowance for variations in the radial distance of each planet, as reflected in its changing size and luminosity), instead of merely adding further homocentric spheres to the extant Eudoxean models.

The incompatibility between physics and astronomy caused little or no complaint during the so-called Middle Ages due to the prevailing lack of intellectual rigor,⁹ but furnished a powerful motivation for Kepler's *astronomia nova sive physica coelestis* and Galileo's simplified (epicycleless) Copernicanism at the beginning of the 17th century. From our modern standpoint it is worth emphasizing that no such incompatibility existed between Eudoxean and epicyclic astronomy. The latter included in its models data which the former ignored and was therefore able to predict—within margins of error that seemed acceptable at the time—changes in radial distance that Eudoxus was unconcerned by. That made them probably better for their purposes, but did not detract from the validity of the Eudoxean scheme for its own purpose of predicting the declination and right ascension of each planet at a particular time. Only the intemperate aspiration to a God's eye view of things can suggest that a mathematical model of phenomena is *better* only if it is *closer to the truth*, an elusive property that is certainly not secured merely by bearing in mind a broader spectrum of data. What Poincaré [29, 215] wrote about the geometries of Euclid and Lobachevsky can generally be said of all mathematical models: one of them cannot be *true* than another one; only more *apt*,¹⁰ in a particular context. Each model enables the derivation of certain statements from certain assumptions. If a phenomenal situation—a “fragment of the world”—does not comply with those statements, neither does it meet those assumptions. But this means only that the said situation is not modeled by that model. Falsehood taints only the claim that it does.

This noncommittal character of models is clearly spelled out at the beginning of the Third Day of Galileo's *Discorsi* when uniform accelerated motion is put forward as a candidate for the mathematical representation of free fall near the surface of the

⁸Compare *Republic* VII, 529b1, 530a3-b3 with *Leges* VII, 822a4-8.

⁹On the downfall of intellect in civilized Europe after the 2nd century B.C. see [31].

¹⁰*Idoine* was Ferdinand Gonseth's French term for it; in German, Einstein said *brauchbar*, i.e. 'usable' [11, 117]. On 'usability' or *Brauchbarkeit*, see also [19].

Earth. Sagredo readily accepts “our author’s” arbitrary definition of uniform accelerated motion and whatever theorems might be derived from it by sheer logic. He only doubts “whether this definition, conceived and accepted in the abstract, is adapted to, agrees with and is verified by that kind of accelerated motion that is carried out by heavy bodies as they fall naturally” (*se tal definizione concepita, e ammessa in astratto, si adatti, convenga, e si verifichi in quella sorte di Moto accelerato, che i gravi naturalmente descendentì vanno esercitando*—Galileo Galilei [13, 158; 14, 8: 198]. To which, after some discussion about the cause of free fall, which Galileo’s spokesman Salviati says it would be untimely to go into,¹¹ he responds as follows:

For the present it suffices our author that we understand that he wants here to inquire and demonstrate some features (*passioni*) of a motion so accelerated (whatever be the cause of its acceleration), that the momenta of its speed go increasing after its departure from rest in the same most simple proportion in which the continuation of time increases, that is to say, that equal increments of speed accrue in equal times. And if it is found that the properties (*accidenti*) which will be demonstrated are verified in the motion of naturally falling and accelerated heavy bodies we may deem that the definition assumed comprises such motion of heavy things, and that it is true that their acceleration goes on increasing as the time and the duration of the motion grows.

(Galileo Galilei [13, 163; 14, 8: 202s])

When combining simple motions—rotations of constant angular velocity—to account for the successively observable positions of each planet on the celestial sphere, Eudoxus did not have to neglect any sensory data. (He did ignore variations of apparent size, but these are relevant to radial motion, which lied outside his field of inquiry.) To model free fall on the surface of the Earth as uniformly accelerated motion, Galileo had to leave out the pervasive presence of surrounding air. Once this idealization is granted, the shape and material composition of each body turn out to be irrelevant. Nancy Cartwright [4] famously said that this is “how the laws of physics lie”, but, to my mind, it would be fairer to say that this is their way of being true. For the validity or—*sit venia verbo*—the *truth* of a modern scientific concept does not consist in its *adequacy* or agreement with the essence of things as planned and created by God; but in its *aptness*, that is, its proven suitability for articulating our grasp of situations of interest and enabling successful predictions about their subsequent development.

In this respect, the advent of modern scientific thinking in the 17th century brought about a decisive change. The *kinematic*, purely descriptive models proposed by pre-modern mathematical astronomy, from Eudoxus to Copernicus and Tycho Brahe, represented each planet as moving along a continuous curve, each point of which stood for an instantaneous position of the planet, close in space to the positions coming right before and after it. From the postulated trajectory, the astronomer can infer the position of the planet at any particular moment. But the time evolution of the planet

¹¹“Non mi par tempo opportuno d’entrare al presente nell’investigazione della causa dell’accelerazione del Moto naturale” [13, 163; 14, 8: 202].

along the trajectory must be assumed; it does not follow from the present state of the planetary system at a particular time. In the modern, so-called *dynamic* models, employed in astronomy since Newton but introduced in terrestrial physics by Galileo, the present state of the represented system at each moment is conceived and described in such a way that its past and future states follow from it with logical certainty, provided that the model aptly represents the past, present and future of the system. If the present state of the modeled process is what a dynamic model assumes it to be, then its future will follow *with necessity*. This can be readily seen by reformulating Galileo's definition of uniform accelerated motion using Leibniz's symbolism for the differential calculus. Uniformly accelerated motion, as defined by Galileo, satisfies the differential equation

$$\frac{d^2s}{dt^2} = g \quad (1)$$

where s is the distance traveled by a uniformly accelerated body B and g is a constant. It follows that the speed v of B at any time t is proportional to t :

$$v = \frac{ds}{dt} = gt + k \quad (2)$$

where the constant of integration $k = 0$ if B started from rest. In this case, the definite integral

$$\int_0^T v dt = \int_0^T ds = \frac{1}{2}gt^2 \quad (3)$$

yields the full distance that B has traveled, along a straight line, at any instant $T > 0$, which may indeed still be future right now. This result is *necessary* for every assigned time T if the motion is an exact realization of the model.

One does not usually resort to the stringent concepts of mathematical analysis for the sake of explaining such a simple model, which had already been designed without them by Galileo and three centuries earlier by the Mertonians. Yet I dare say that before Newton and Leibniz invented the calculus in the late 1600's, or indeed before mathematicians from Cauchy to Dedekind succeeded in elucidating it in the 1800's, it was not easy to see how the fugacious present could have a grip on any other instant, let alone a future one. Indeed I would say that it was not possible to see this while change was not understood as a way of actually existing. Aristotle timidly foreshadowed this understanding within the Eleatico-Platonic tradition when he defined motion as the actuality of potential being as such (ἡ τοῦ δυνάμει ὄντος ἐντελέχεια, ἥ τοιοῦτον, κίνησις ἐστίν; *Physica*, III, 1; 201^a10–11; in [1]); but change did not obtain a proper place in European science and philosophy among the full-fledged modes of being until Descartes dismissed Aristotle's definition as rubbish,¹² and classed motion as an actual state of things. On this

¹²“Does it not appear that those who say that *motion*—a reality most familiar to everyone—is the act of potentiality insofar as it is potential are uttering magical words, prompted by a hidden force, beyond the grasp of human intelligence? Who understands these words? Who does not know what motion is? And who would not admit that those authors seek a difficulty where there is none?” (“At verò nonne videntur

view, velocity and acceleration may well be present, actually operative attributes of the mobile, and not just averages of earlier performances. And this is precisely how they are conceived under the mathematical notion of the time derivative, defined as the limit to which such averages converge as the time intervals over which they are taken converge to the current instant. Generally speaking, a model that characterizes the present condition of a physical system by state variables linked together by a sufficient set of differential equations fixes thereby the value of those variables for all times and therefore determines with necessity the past and future evolution of the system if and while it can pass for a realization of that model.

After four fruitful and progressive centuries of mathematical physics there is no doubt that we can construct and sustain models of phenomena that constrain their future development within narrow paths. However, this does not imply that all future events are determined beforehand and could be foreseen by someone who knew the past well enough. Nothing warrants this metaphysical conceit, which was nevertheless upheld by Laplace and other 19th-century scientists even after they had given up the religious faith that might have made it plausible.¹³ Their overconfidence was nourished by Newton's intrepid yet incredibly successful extension of Galileo's theory of free fall to the Moon, the entire Solar System and the whole universe. Setting aside Galileo's moratorium on the investigation of the causes of accelerated motion (see the quotation in n. 11), Newton imputed every such motion to an external force acting on the accelerated body in the direction of the acceleration, whose magnitude depends on the circumstances of each case but is always proportional to the said body's "quantity of matter" or *mass* and to the acceleration itself. In particular, a particle *A* with mass m_A will fall freely towards a particle *B* with mass m_B with the variable acceleration $\frac{d^2\mathbf{r}}{dt^2}$ that can be calculated from

$$m_A \frac{d^2\mathbf{r}}{dt^2} = -G \frac{m_A m_B \mathbf{r}}{r^3} \quad (4)$$

where \mathbf{r} is the radius vector drawn from the center of mass of the $A + B$ system to particle *A*, and G is a proportionality constant. It is not hard to see (1) as representing the special case in which *A* falls over such a small distance that \mathbf{r} can be considered constant both in magnitude and in direction. Newton reached (4) through the mathematical modeling of planetary phenomena and showed by calculation that a "little moon" (*lunula*)¹⁴ moving in accordance with it close to the highest mountains would behave

illi verba magica proferre, quæ vim habeant occultam et supra captum humani ingenij, qui dicunt *motum*, rem uniuersicquæ notissima, *esse actus in potentiâ, prout est in potentiâ*? quis enim intelligit hæc verba? quis ignorat quid sit motus? et quis non fateatur illos nodum in scirpo quæsiuisse?"—Descartes, *Regula XII*; AT, 10: 426; this passage is omitted in the popular English translation of the *Regulæ* by Elizabeth Anscombe, available in several websites).

¹³The plausibility, nay, the inevitability of global determinism for a faithful Christian was trenchantly expressed by Jacques Bernoulli [2, 227]: "That anything should be uncertain or indeterminate in itself, and by its nature, we can so little conceive as we can conceive that the Author of nature at once created it and did not create it. For everything that God has made, He has also determined, by the very act of making it." ("Sed quicquam in se & sua natura tale [i.e. incertum et indeterminatum] esse, non magis à nobis concipi posse, quàm concipi potest, idem simul ab Auctore naturæ creatum esse & non creatum: quaecunque enim Deus fecit, eo ipso dum fecit, etiam determinauit".)

¹⁴See [22, 398].

exactly like a Galileian projectile reaching such height with the appropriate speed. Flying on the wings of his powerful and venturesome *Regulae philosophandi*¹⁵ he extended (4) to the whole material universe. This entails that any (quasi)closed n -body system is governed by a system of simultaneous differential equations of the same form as (4), defined by letting $\langle A, B \rangle$ range over all pairs of bodies. Poincaré [30] proved that such a system does not admit generic closed exact solutions for $n > 2$; but approximate solutions were achieved through the progress of mathematical analysis and can demonstrably be improved indefinitely. This result, added to the smooth imbedding of Galileo's theory of free fall in Newton's theory of universal gravitation, and the apparently successful meshing together of the latter with the new theories of other physical forces during the 19th century, fortified the confidence of scientists in the eventual convergence of all sciences in a single, coherent, deterministic Theory of Everything. This dream remained alive even for such a lucid scientist-philosopher as Charles Saunders Peirce,¹⁶ notwithstanding his farsighted "tychistic" dismissal of determinism; but has been vanishing slowly but inexorably since 1900, after the so-called quantum and relativistic revolutions. On the one hand, the generally accepted theories of microphysics conceive every gradually desintegrating chunk of radioactive material as an irreducibly random chance setup; on the other, the increasing wealth and precision of experimental and observational results forced the replacement of the great physical theories of Newton, Euler, Lagrange, Hamilton, Jacobi, Maxwell, Boltzmann, Lorentz, etc. with the new theories of Einstein, Heisenberg, Schrödinger, Dirac, Weinberg, etc., whose predictions agree with those of the

¹⁵Of the four "rules by which to do philosophy" printed in the third edition of *Principia* [22, 387–389], only the first two appeared in the first [20, 402], where they were labeled 'hypotheses', together with other seven statements, five of which was subsequently relabeled 'phenomena', another one was renumbered "Hypothesis I" and placed after Prop. X of Book III, and still another was deleted. Rule III was added in the second edition [21, 357]. It is, however, clear to me that Newton put all four to work for him from the outset, including the epistemically innovative and audacious Rule IV, first published in [22, 389]: "In experimental philosophy, any propositions gathered by induction from phenomena shall be held to be true—either accurately or to the best available approximation—notwithstanding any contrary hypotheses, until other phenomena occur by which they may either be made more accurate or liable to exceptions" (*In philosophia experimentalī, propositiones ex phenominis per inductionem collectae. non obstantibus contrariis hypothesibus, pro veris aut accurate aut quamproxime haberi debent, donec alia occurrerint phenomina. per quae aut accuratiores reddantur aut exceptionibus obnoxiae*). Here the word *phenomena* does not, as in some English translations of Kant, stand for mere sense appearances (which of course are not "held to be true" *accurate*, let alone *quamproxime*), but for percepts methodically gathered under concepts.

¹⁶"[A]ll the followers of science are fully persuaded that the processes of investigation, if only pushed far enough, will give one certain solution to every question to which they can be applied. [...] They may at first obtain different results, but, as each perfects his ethod and his processes, the results will move steadily together toward a destined centre. [...] Different minds may set out with the most antagonistic views, but the progress of investigation carries them by a force outside of themselves to one and the same conclusion. This activity of thought by which we are carried, not where we wish, but to a foreordained goal, is like the operation of destiny. No modification of the point of view taken, no selection of other facts for study, no natural bent of mind even, can enable a man to escape the predestinate opinion. This great law is embodied in the conception of truth and reality. The opinion which is fated to be ultimately agreed to by all who investigate, is what we mean by the truth, and the object represented in this opinion is the real" (Peirce [25], in [27, 273]). "The real, then, is that which, sooner or later, information and reasoning would finally result in, and which is therefore independent of the vagaries of me and you" (Peirce [24], in [26, 239]).

former in the cases and within the margins of error in which these were successful, but whose conceptual framework and overall understanding of the same phenomena is profoundly at variance with theirs.

Thus, for example, Einstein's General Theory of Relativity (GTR) predicts the motion of all the larger planets with the same great accuracy that Newton's Theory of Gravity (NTG) had achieved by 1913,¹⁷ and also predicts the 43'' secular precession of Mercury's perihelion, which was a thorn in the flesh of prerelativistic astronomy; but it can account moreover for a number of surprising phenomena, discovered in the 20th century, which lie beyond NTG's reach: the so-called gravitational retardation of clocks; the bending of starlight grazing the Sun and, generally, the phenomenon of gravitational lenses; the formation of neutron stars through gravitational collapse; the loss of energy by binary stars through the emission of gravitational waves; the steady flight away from us of the galaxies that surround us; the current very low temperature of background thermal radiation, and whatever evidence there is for the existence of black holes. However, Einstein's GTR can only do this because he rethought the very notion of gravity, which he understands not, like Newton, as a force exercised by every pair of bodies on each other in accordance with (4), but as the manifestation of the inertial, i.e. unforced motion of matter on a curved spacetime governed by his field equations, viz.,

$$R_{jk} - \frac{1}{2}Rg_{jk} + \lambda g_{jk} = -\kappa T_{jk} \quad (0 \leq j, k \leq 3) \quad (5)$$

where the g_{jk} and the R_{jk} are, respectively, the components of the Lorentzian metric and the Ricci tensor characteristic of the spacetime geometry, R is the curvature scalar, the T_{jk} are the components of a tensor representing the distribution of matter and non-gravitational energy, and the Greek letters stand for universal constants. It is in a sense ironic that now that so many "little moons" have been strewn by missiles all over the sublunar space, confirming Newton's vision in a way that would have justifiably filled him with endless pride, we must introduce relativistic corrections in their equations of motion to achieve the predictive precision required for employing them, e.g., in the Global Positioning System.

It is generally acknowledged that GTR, which is currently our best theory of physics in the large and very large, is incompatible with the quantum theories which are still our best theories of microphysics. Quantum theories have entered more than once into profitable *mésalliances* with GTR, e.g., to explain the background radiation discovered by Penzias and Wilson [28] or in Hawking's theory of the evaporation of black holes [15]. But all the strenuous efforts spent during the last thirty years to produce a quantum theory of gravity that accounts for, absorbs and supersedes GTR, have hitherto failed to do so. Nobody questions that such a unified scheme would be a source of enormous aesthetic and intellectual satisfaction. Still, if such a scheme be

¹⁷According to Einstein [10, 1249], Newton's simple laws of the motion of a mass-point and of the interaction of two gravitating mass-points had proved themselves so exactly accurate that "from the standpoint of experience there is no decisive ground for doubting their strict validity" ("Diese Gesetze haben sich derart exakt erwiesen, daß vom Standpunkte der Erfahrung aus kein entscheidender Grund vorliegt, an der strengen Gültigkeit derselben zu zweifeln").

found, we ought nevertheless to expect newly open perspectives to put new situations on view that the freshly available concepts and models will be unable to account for. Does this imply that we must despair of human knowledge? I, for one, would rather retain ‘knowledge’ as a live, applicable word, and not just the alternative name for the empty set, which it would be if we impose on its referent unattainable demands of infallibility and totality.

Summing up: I have no reason to doubt that the Sun will *necessarily* rise tomorrow, in the Newtonian or in the Einsteinian scheme of things, or in any other scheme that can account for the past—as we know it—better than they do. To rest assured of it, I must indeed assume that the quasiclosed dynamical system to which the Sun and the Earth belong will not tonight be disturbed by a major intervention from outside. But again, I have no reason to expect that it will be so disturbed; although I admit that we might have big surprises on this issue while we know practically nothing about the properties and the distribution of “dark energy”. However, such surprises remain possible not because the future is not tied to the past and therefore unknowable, as Humean empiricists proclaim, but because we are vastly and deeply ignorant of the present.

The necessity of future development, conceptually conveyed by differential equations assumed to hold *now*, is easily understood but, perhaps for this very reason, it has an air of fiction. To get a more persuasive feel of the bonds tying the future to the present one must free oneself of the age-old philosophical disjointing of contemplation and action, *theoria* and *praxis*, dismiss Kant’s handless *Vernunft*, and pay attention to the primary role of concepts in the conscious launching and performance of muscular operations: walking, running, sitting, pointing, seizing, holding... Inevitably, to *be* what it *is*—i.e., what was decided—the operation in question must proceed in a fashion known in advance (unless indeed it is thwarted). Through the original, thorough imbrication of human concepts and percepts with human interests and projects, the reality of our knowledge of the future, though not of course its infallibility, is secured.

The examples cited display scientific models as conceptual nets with tight inner cohesion and a light-handed grip on phenomena. In their crazy craving for certainty, epistemologists tend to forget that conceptual grasp, though fallible, can only be corrected in the light of a different conceptual grasp: Conceptualizations can only be judged by their peers. On the other hand, it is a pleasant fact of life that models proposed by sensible people for modeling passably well defined situations usually agree with the aspects under consideration, within the allowed margin of inaccuracy.

1 Supplementary Remark on de Finetti’s Philosophical Position

Reading (while I wrote this paper) Bruno de Finetti’s brilliant posthumous book *L’invenzione della verità* [7], I realize that he stood much closer to the reconstructed or “enlightened” Kantianism I favor, than his derogatory remarks about Kant would suggest (cf. [6, 2: 201]). I have the feeling that he failed to endorse it not due to lack of insight into the actual makeup of human knowledge, but rather because he did not venture to go the whole way in his rejection of Platonic foundationism. He is still

under the illusion that philosophy can start otherwise than *in medias res* (“all’inizio il lettore non dovrà supporre di avere preformato nessun concetto”—[7, 97]), and this constrains him—like Hume—to hypostatize sensations. Logic is “invented” according to him by fixing our attention on sensations (“*Io ho delle sensazioni, e per fissare su di esse le idee trovo utile pensare degli ‘eventi’ da distinguere in ‘veri’ e ‘falsi’*”—[7, 100]). With the aid of logic, one proceeds “to consider sensations with respect to their content, in order to coordinate them... in a coherent system which will be (*sarà*) the ‘world’ or ‘reality’” ([7, 123]; four lines below this, Carnap [3] is appositely invoked).

To clarify the meaning and scope of the assertions which—for the sake of contrast—I attributed to de Finetti at the beginning of this paper, I have collected here a few quotations: “*Prevision* [...] does not involve guessing anything. It does not assert—as prediction does—something that might turn out to be true or false, by transforming [...] the uncertainty into a claimed, but worthless, certainty. [...] Faced with uncertainty, one feels, and You feel too, a more or less strong propensity to expect that certain alternatives rather than others will turn out to be true [...]” [6, 1: 71]. “Uncertain things remain uncertain, but we attribute to the various uncertain events a greater or lesser degree of that new factor which is extralogical, subjective and personal (mine, yours, his, anybody’s), and which expresses these attitudes. In everyday language this is called *probability* [...]. *Prevision*, in the sense we give to the term [...], consists in considering, after careful reflection, all the possible alternatives, in order to distribute among them, in the way which will appear most appropriate, one’s own expectations, one’s own sensations of probability” (1: 72). “The probability $P(E)$ that You attribute to an event E is [...] the certain gain p that you judge equivalent to a unit gain conditional on the occurrence of E ” (1: 75). “Everything that does not reduce to a simple finding (*constatation*), to an isolated historical truth, everything that counsels us for the future, even the belief that in leaving our room we shall see as on other days the same streets and the same houses in their same places, all that constitutes a judgment of probability that is based, perhaps unconsciously and indistinctly, on the principles of the calculus of probability” [5, 65]. “The notion of ‘cause’ [...] flows from the same subjective source as every judgment of probability; this explanation would be the true logical translation of the conception of ‘cause’ advocated by David Hume, which I consider the highest peak attained by philosophy” (Ibid.).

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